Application of Clément's first formula to an arranged-schedule secondary canal

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Abstract

Clément's first formula was initially intended to determine discharge in on-demand pressurized irrigation networks; nevertheless, it can also be applied to canal systems because the irrigation process is similar. Discharges at a secondary branch of the Aragon and Catalonia canal (Spain). with an arranged schedule of 24-hour duration and 1-day delay, were analyzed. Since the canal was high-capacity and, as such, no request restrictions were necessary, Clément's formula could potentially simulate discharges in this case. Canal's intakes were modeled assuming normally distributed variables represented by their average and standard deviation. Clément's first formula accurately fit the observed cumulative probability curve of canal discharge at different sections. The relationship between the average intake's discharge and the product of the specific continuous discharge by the service area was also assessed. Clemmens' arranged delivery schedule equation does not apply to this case because the canal delivers pressurized irrigation and high turnout opening probabilities occur.

Keywords: irrigation networks, canal capacity, arranged schedule, Clément's first formula, on-demand delivery, secondary canal

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Introduction

When flexible irrigation schedules are adopted in an irrigation network, water is used more efficiently (Replogle and Merriam, 1980). Among these schedules, the on-demand method offers the most flexibility, though high canal capacity and costly automatic structures are essential. The need for automated gates, a specialized workforce, sufficient water resources, and highly informed farmers are among the limitations of the on-demand method. Alternatively, utilization of the on-request method can be employed within existing irrigation networks by making slight alterations to network management (Burt, 2011).
One of the most important issues with an on-demand irrigation system is correctly calculating the discharge flowing into the network (Lamaddalena et al., 2000). Statistical methods are traditionally used to solve this problem. One of the models used to determine pipe capacity in on-demand irrigation systems is the probabilistic approach introduced by René Clément in 1966, generally referred to as "Clément's first formula," in which the number of farmers that simultaneously demand water is approximated using normal distribution (Lamaddalena et al., 2000).
Assuming homogeneous hydrants, the number of operating hydrants (x) is a random variable having a binomial distribution with a mean ($R\,p$) and variance ($R\;p\;q$), where R is the total number of hydrants, p is the probability that a hydrant is open (elementary probability), and q is the probability that a hydrant is closed. When R is large enough, the binomial distribution approximates Laplace-Gauss normal distribution and the standard normal U_{p_q} can be obtained using Eq. 1 (Lamaddalena et al., 2000):

$$U_{Pq} = \frac{x - Rp}{\sqrt{Rpq}} \qquad (1)$$

Equation 1 can be solved for x and multiplied by d, the hydrant's nominal discharge. Next, the section's discharge is obtained from Eq. 2 (Clément, 1966).

$$Q_{Clement} = Mean + U_{Pq}\sqrt{Variance} \qquad (2)$$

When hydrants have different discharges (d_i), the section's discharge can be calculated using Eq. 3 (Clément, 1966):

$$Q_{Clement} = \sum_{i=1}^{R} p_i d_i + U_{Pq}\sqrt{\sum_{i=1}^{R} p_i(1 - p_i)d_i^2} \qquad (3)$$

Where, $Q_{Clement}$ is the total discharge at a generic section in $l\ s^{-1}$; p_i is the probability that a hydrant is open (elementary probability); q_i is the probability that a hydrant is closed; d_i is the nominal discharge for each hydrant in $l\ s^{-1}$; P_q is the cumulative probability of the simultaneous operation of hydrants; and U_{Pq} is the standard normal variable corresponding to P_q.
Clément introduced a second formula designed using a specific type of Markov chain referred to as birth and death processes. Clemmens (1986) said that this model is better adapted to irrigation events on a canal because it is based on congestion probability, not time-based frequency. Lamaddalena (2000) reported small differences between the two equations.

Due to the higher mathematical complexity of the second formula, most designers use the first.

Clément's formula can also be applied to canals with turnouts as they are akin to a pressurized pipe with hydrants. The turnout is opened for some given amount of time during which constant discharge is delivered. Researchers have also applied this formula to surface irrigation systems. Clemmens (1986) investigated the impact of network schedules on canal capacity. Simulations were performed assuming surface irrigated plots, certain crops, specific soils, and 20 years of weather data. The author compared the simulation results with those from Clément's formulas. The discharges from Clément's second formula and a hypothetical arranged schedule were in good agreement. All results are presented in a dimensionless form to be more generalizable. The flow rate of a canal (Q) is referenced to the average turnout design flow rate (Q_t) to give a relative flow rate. Similarly, the size of a canal service area (A) is referenced to the rotation unit size (A_t), which provides the relative service area (A_n).

$$Q_n = \frac{Q}{Q_t} \ ; \ A_n = \frac{A}{A_t} \qquad (4)$$

Clément's first formula in dimensionless form, concerning homogeneous plots is (Clemmens, 1986):

$$Q_n \ = \ A_n + U_{p_q} \sqrt{A_n \, (1-p)} \qquad (5)$$

Clemmens (1986) observed that the results of Clément's first and second formulae converge for A_n values greater than 5. Moreover, when the dimensionless area (A_n) is less than 1, Q_n values are also less than 1, which is not feasible for surface irrigation. This arises from the fact that Clément's formula is a statistical equation and, as such, does not consider specific features of the system to which it's being applied (Bonnal, 1963; Monserrat et al., 2013). Clemmens (1986) proposed Eqs. 6 and 7 for arranged-schedule canals:

$$Q_n = 1.6\, A_n + 1 \ ; \ A_n < 1 \qquad (6)$$
$$Q_n = A_n + 1.6 \ ; \ A_n > 1 \qquad (7)$$

Anwar et al. (2006) propose that arranged scheduling is a type of on-demand schedule. They create an index of relative timeliness (R_t) to distinguish between different arranged schedules. This index assumes a value of 1 for an on-demand system and 0 for a fully arranged demand system. Using an integer programming technique, they determine the canal capacity for different timeliness indexes and compare their results with those from Clemmens' arranged schedule formula. Anwar et al. (2006) analyzed a case where they assume $A_n \ = \ 1$ when, in fact, A_n was = 2.67. The computed Clemmens' dimensionless arranged discharge (Eq. 7) for this area is Q_n = 4.27, which is close to what Anwar obtained with a 1-day delay time and 90% service level. This means that Anwar's results are similar to Clemmens', unlike what Anwar said. Anwar and Haq (2016) test another mathematical technique, genetic algorithms, to organize farmer demands in a way that minimizes canal discharge and maximizes efficiency.

Few papers with real canal delivery data have been published. In this study, discharges from a secondary canal were analyzed and compared with the results from Clément's first formula, which was selected for its simplicity.

Materials and Methods

Data from a secondary canal of the Aragon and Catalonia canal in Spain were used in this study. It is 15-km long with nine intakes, each of which supplies a small reservoir. From each intake, a local water user association distributed the water to each user. The secondary canal was managed according to an arranged schedule. Each associate could demand water once per day. The time from demand to delivery was one day, the maximum unit flow rate was 0.6 l s^{-1} ha^{-1}, the intake frequency was variable, and the delivery duration was fixed to 24 h. All intakes and regulatory structures within this network were operated by the Supervisory Control And Data Acquisition (SCADA) system.
Data gathered from each intake were daily delivery discharge values from July 1 to July 31 for four years (2016 to 2019). Therefore, four replicates can be considered. The intake discharge notation is: Q_{i_j} where j is the intake number (1 to 9). The calculated sample error related to the sample size is about 7%.
During the study period, the supplied discharge was the same as the requested discharge so there was no constraint to this variable.
A canal's intake cannot be modeled like a turnout (with d_i and p_i) because it is the head section of a tertiary canal and usually, discharge is continuously flowing. According to Clément's first model, the intake discharge is a normal variable characterized by its mean and standard deviation. As such, the discharge at section j (Fig. 1) of a secondary canal can be calculated from Eq. 8.

$$Q_{c_j} = \sum_{1}^{j} \bar{Q}_{i_j} + U_{P_q} \sqrt{\sum_{1}^{j} Var(Q_{i_j})} \qquad (8)$$

Where Q_{c_j} is the canal discharge at section j; $\bar{Q}_{i_j}$and $Var(Q_{i_j})$ are, respectively, the average intake discharge and its variance.

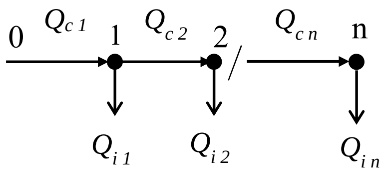

Figure 1. Secondary canal sketch: 1, 2, n, canal intakes, Q_c ,discharge at canal section; Q_i , Canal intake discharge.

An ex-post assessment of this equation can be carried out using the intake's discharge data. In that case, the average and variance of intake's discharges can be calculated and then the results of Eq. 8 can be compared with real data. This will determine whether the normal function fits real data.
Moreover, an ex-ante assessment can be done considering weather data, and the crop's areas. From this data, the specific continuous discharge rate can be calculated then average canal discharge can also be obtained by Eq. 12, which is deduced below.
An intake discharge ($Q_{i\,j}$) can be calculated by applying Clément's first formula (Eq. 3), considering that an intake supplies T plots. For a service level of 50%, then U = 0, so

$$Q_{i_j} = \sum_{i=1}^{T} p_i d_i \qquad (9)$$

From Lamaddalena (2000), the turnout opening probability can be expressed by

$$p_i = \frac{q_s A_i}{d_i\, r} \qquad (10)$$

Where q_s is the specific continuous discharge $l\, s^{-1}\, ha^{-1}$; d_i is the nominal discharge for each turnout in $l\, s^{-1}$; A_i is the plot area; r is the coefficient of utilization of the network, which is the ratio between actual network available time to 24 hr; in our case $r = 1$.

Then,

$$Q_{i_j} = q_s \sum_{i=1}^{T} A_i = \bar{Q}_{i_j} \qquad (11)$$

Substituting Eq. 11 into Eq. 8 with the same assumptions (50% service level),

$$\bar{Q}_{c_1} = q_s \sum_{j=1}^{9} (A_T)_j = q_s\, A_{T\,c} \qquad (12)$$

Where ($\bar{Q}_{c_1}$) is the average discharge at the canal head and $A_{T\,c}$ is the total crop area supplied by the canal.

The FAO-56 method (Allen et al., 1998) was used to determine the specific continuous discharge of the study area. Table 1 shows the crop areas for each year. Reference evapotranspiration (ET_0) and daily rainfall values (P) were gathered from a nearby weather station. The crop coefficients (k_c) were taken from FAO-56, and the effective rainfall (P_e) was estimated as 0, if $P < 4$ mm, and 0.4 P, if $P > 4$ mm (Cots, 2011). The global irrigation efficiency was estimated to be 0.89.

The variance of intake's discharges can be calculated by the second term of Eq. 3, so turnout values for d_i and p_i are needed.

Table 1: Crops area (ha) by year

Year	Long-cycle Corn	Short-cycle Corn	Alfalfa	Fruit Trees	Total
2016	139.5	187.5	91.1	767.4	1185.5
2017	135.7	169.8	133.9	756.2	1195.6
2018	89.5	204.1	136.1	763.3	1193.0
2019	163.7	152.1	207.8	686.6	1210.2

Results

Figure 2 shows the total discharge delivered at the head of the canal during the study period. In this figure, some extreme values were observed. High discharge levels align with high temperatures in early July 2019 and the sudden reduction on July 9th reflects rainfall.

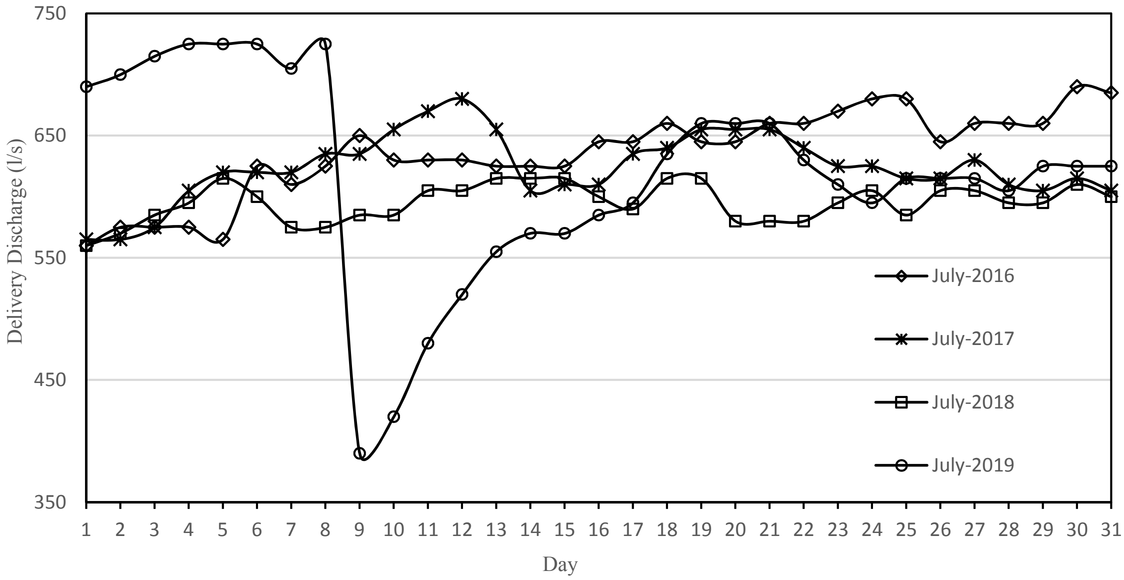

Figure 2. Daily delivered discharge at the head of the secondary canal.

Table 2 shows the average discharge and variance for each intake. The intake's opening probability (p) shows that almost all intakes are always operating except for the fourth one.

Table 2: Average discharge, variance, and opening probability by intake

Intake	Average $\bar{Q}_i$ (l/s)	Variance $(l/s)^2$	p
1	57.8	104.5	1
2	35.6	133.7	1
3	113.0	464.9	1
4	15.2	53.2	0.87
5	37.9	29.7	1
6	32.4	70.5	1
7	52.2	142.3	1
8	165.8	468.4	1
9	108.5	162.6	1

Figure 3 shows the coefficient of variation (CV) for the intakes' discharges. The CV remains nearly constant for discharges over 40 l/s and increases with lower discharges. The average value (0.23) is similar to that observed (0.25) in a pressurized network by Monserrat et al. (2004). The highest CV corresponded to the intake that was closed on some days during the study period.

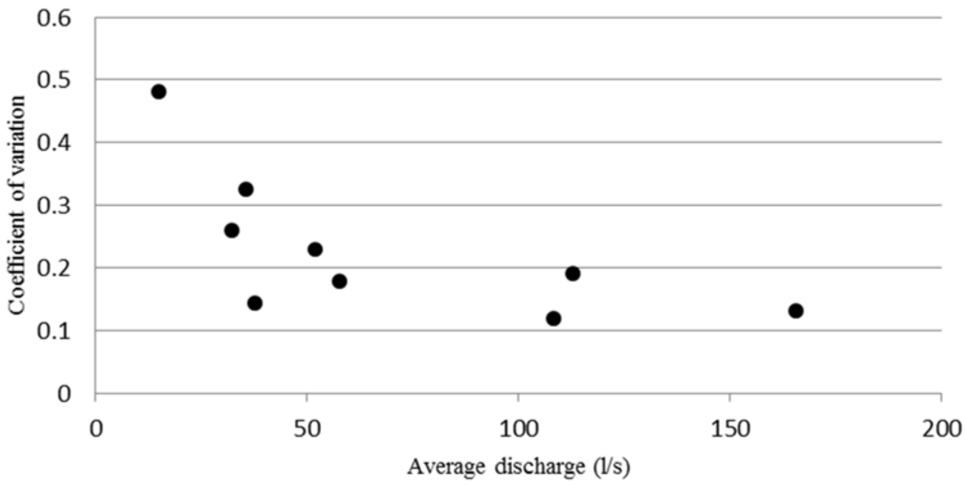

Figure 3: Coefficient of variation of intakes' discharge

Assessment of Clément's first formula

Ex-post assessment

Discharges at the upstream end of the canal were analyzed first. Table 3 presents the main statistics for these data. The negative skewness indicates that the data were slightly concentrated on the right side, and the high kurtosis value indicates that the data were concentrated around the mean. Lilliefors (Kolmogorov-Smirnov) normality test applied to real data gave a $p = 0.1388 > 0.05$, indicating that the real data was normally distributed.

Table 3. Main statistics of the upstream discharge

Mean (l/s)	Std. Dev. (l/s)	Coef. Var. (%)	Skewness	Kurtosis
618.4	49.8	0.08	-1.054	4.832

Clément's results were generated assuming that the discharge amounts at the head of the canal were normally distributed with an average of $\sum_{j=1}^{9} \bar{Q}_{i_j}$ and a standard deviation of $\sqrt{\sum_{j=1}^{9} Var(Q_{i_j})}$ (Eq. 8). The comparison between the discharges determined from Clément's first formula and real data are presented in Figure 4.

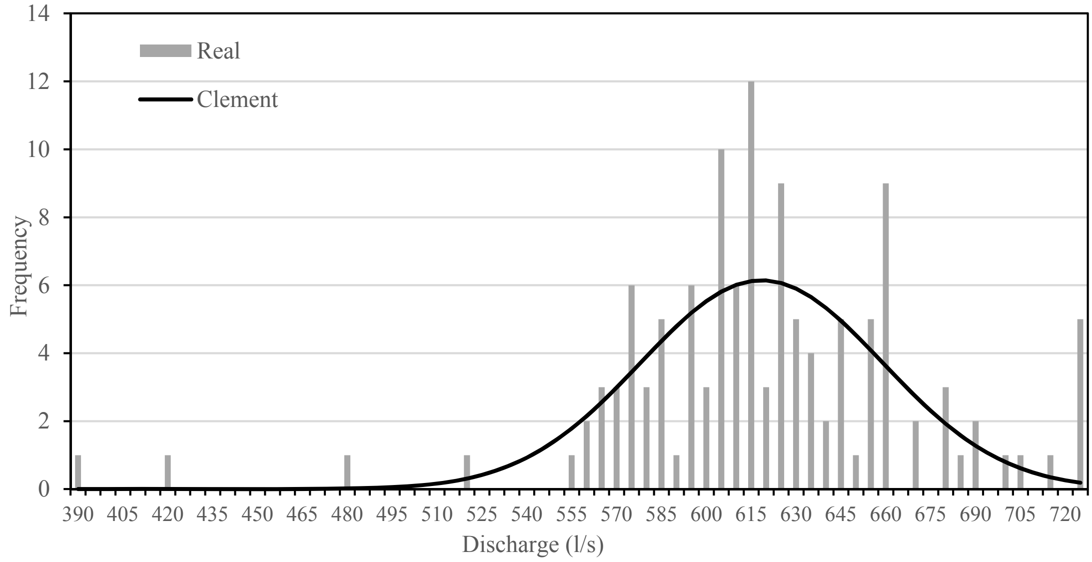

Figure 4. Histogram of real data and the distribution obtained with Clément's first formula at the canal head.

Data trends also indicated that the flow was not constrained by canal capacity because high discharges did not have high frequencies, so it can be considered a quasi-demand schedule. The cumulative probability fitting (Fig. 5) is better than seen in Figure 4 because the sum of the frequencies compensates for any observed differences.

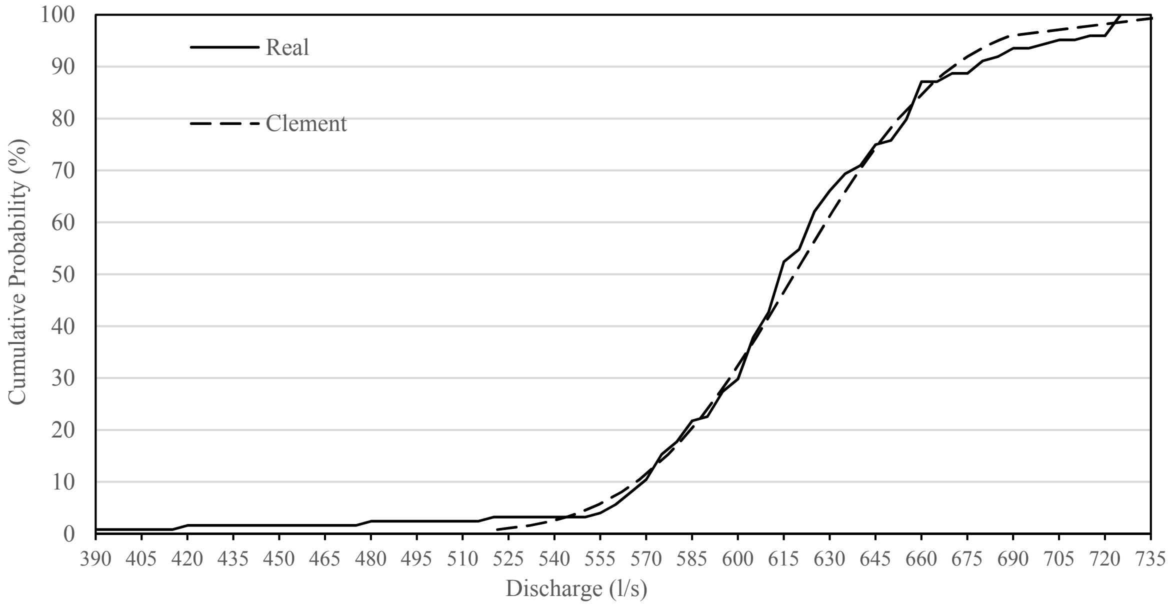

Figure 5: The cumulative probability of the real data and those obtained with Clément's first formula at the canal head.

Clément's formula error was computed using Eq. 13:

$$\text{Error (\%)} = \left(\frac{Q_{Clement} - Q_{Real}}{Q_{Real}}\right) * 100 \qquad (13)$$

The error at the canal head has been drawn for a cumulative probability higher than 50% because under 50% is not practical for irrigation canals. There was good agreement between the real data and those generated using Clément's first formula because the maximum error was less than 4.3% (Fig. 6).

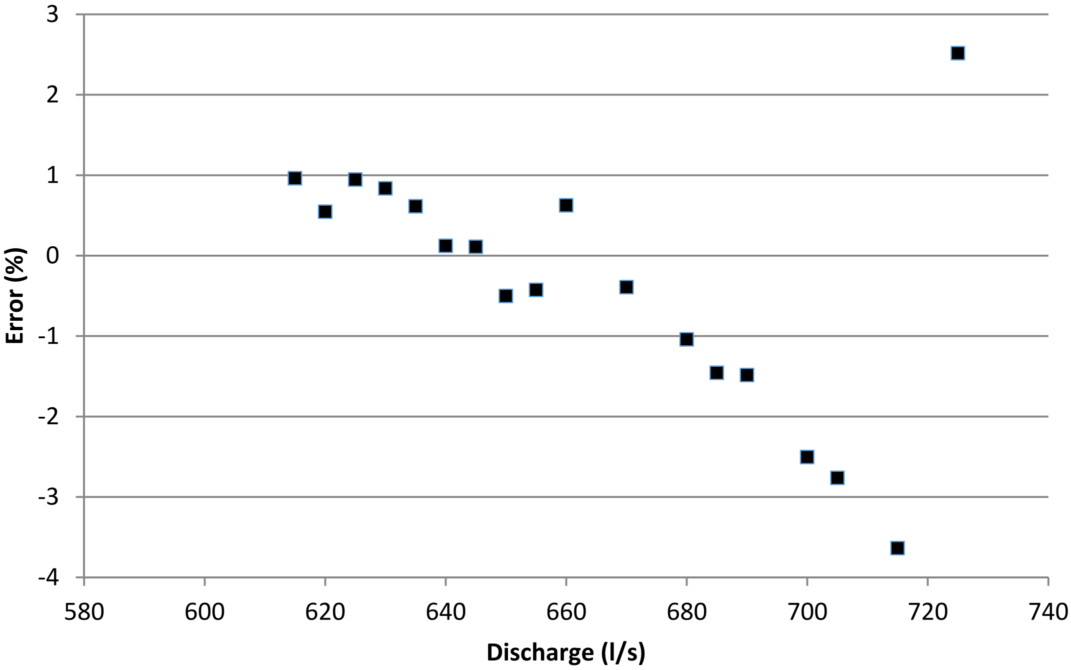

Figure 6: Error of Clément's first formula at the head of the canal for different discharges.

Figure 6 shows that Clément's formula underestimates when the discharge is high and slightly overestimates at low discharges.

In Figure 7, the error for different cumulative probabilities (over 50%) is shown for several sections along the canal. Each section is named for its associated intake. The error increases at sections closer to the end of the canal. Nevertheless, the magnitude of this error is low.

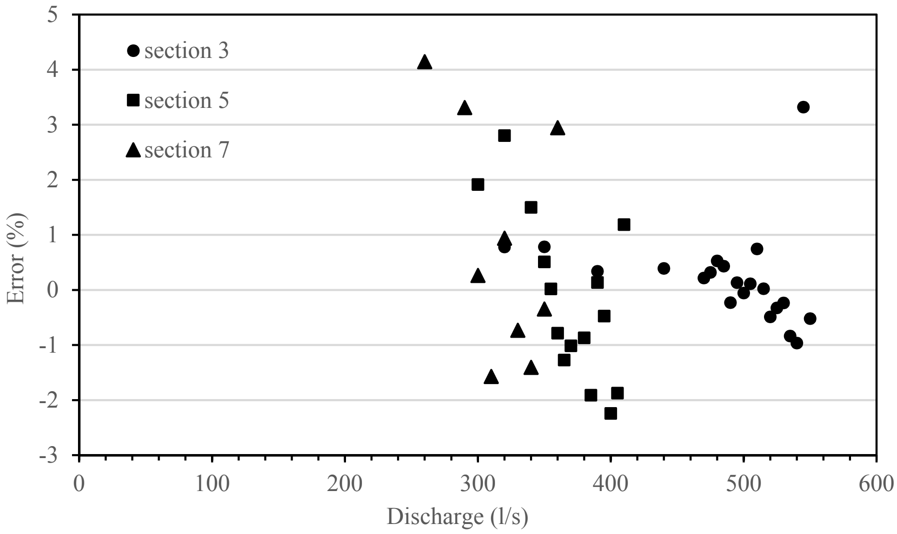

Figure 7: Error of Clément's first formula at several sections of the canal for different discharges.

Ex-ante assessment

Table 4 shows the monthly water balance values and the resulting average required discharge. Figure 8 illustrates the relationship between the observed monthly average discharge at the canal head and the required discharge calculated from the crops water needs. The average error was 3%, which indicates good agreement between both discharges and, therefore Eq. 12, was checked. Discharge variance cannot be calculated because plot data were not available.

Table 4. Monthly components of water balance and required discharge

Year	P (mm)	ETo (mm)	$k_{c\ av}$	ETc (mm)	P_e (mm)	IR (mm)	IR ($l\ s^{-1}ha^{-1}$)	Q_{req} ($l\ s^{-1}$)
2016	1,6	174,02	0,74	128,43	0,00	143,98	0,54	637,0
2017	11,4	167,78	0,73	122,65	1,88	134,78	0,50	601,4
2018	27,8	176,54	0,71	125,87	8,12	137,27	0,51	611,4
2019	41,9	167,27	0,72	119,77	15,08	137,92	0,51	623,1

P, total rainfall; ET_0 , reference ET; $k_{c\ av}$, average crop's coefficient; P_e, effective rainfall; IR, irrigation requirement; Q_{req} , average monthly required discharge.

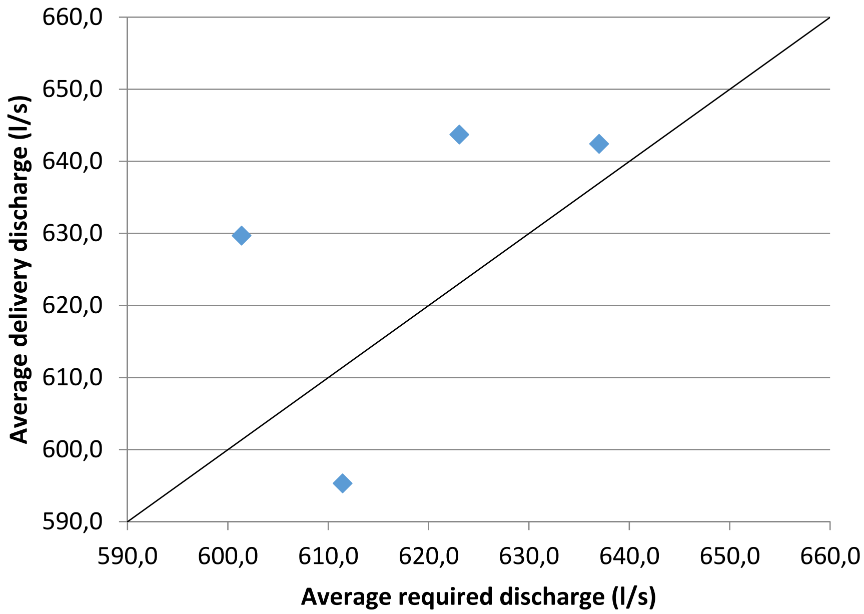

Figure 8. Relationship between measured monthly average delivery discharge and calculated average required discharge for years 2016 to 2019.

Application of Clemmens' arranged delivery schedule formula

Because the studied canal had an arranged schedule, Clemmens' arranged formula (Eq. 7) may be applied and compared with Clément's first formula, which was previously checked. The average intake discharge $(\bar{Q}_{i_{j\,k}}) = \frac{\sum_{j=1}^{9} \bar{Q}_{i_j}}{9} = 68.7\,\frac{\mathrm{l}}{\mathrm{s}}$ was taken as the characteristic discharge, Q_t. The characteristic area was obtained by $A_t = \frac{Q_t}{q_s}$. The average specific continuous discharge q_s can be determined by table 4, resulting in $q_s = 0.52\ \mathrm{l\ s^{-1}\ ha^{-1}}$ and by applying the A_t definition, $A_t = 132.1$ ha.

From the A_t definition, $A_t = \frac{Q_t}{q_s}$ and Q_t definition, $Q_t = \bar{Q}_{i_j}$ it can be deduced that,

$$A_t = \frac{\sum_{j=1}^{9} A_{T\,j}}{9} \qquad (14)$$

Which means that A_t can be computed from the average intake's crop area independent of any discharge.

Figure 9 presents a comparison between Clemmens' arranged formula (Eq. 7) and Clément's first formula for a service level of 95%. The arranged formula gave greater values for the entire range, these are unexpected results because the degree of freedom is higher in the on-demand formula and thus, the discharges should be higher. To explain this inconsistency, a fictitious tertiary canal was supposed and Eq. 5 was applied for two arbitrary probabilities, $p = 0.08$ and $p = 0.9$, for a 95% service level. The results for both probabilities and the arranged formula are shown in Figure 10. Clément's demand formula for $p = 0.9$ gave lower values than the arranged formula, as in the case of the secondary canal (Fig. 9). This can be explained because Clemmens' formula was obtained for surface irrigation conditions at which low opening probabilities occur ($p_{av} = 0.06$).

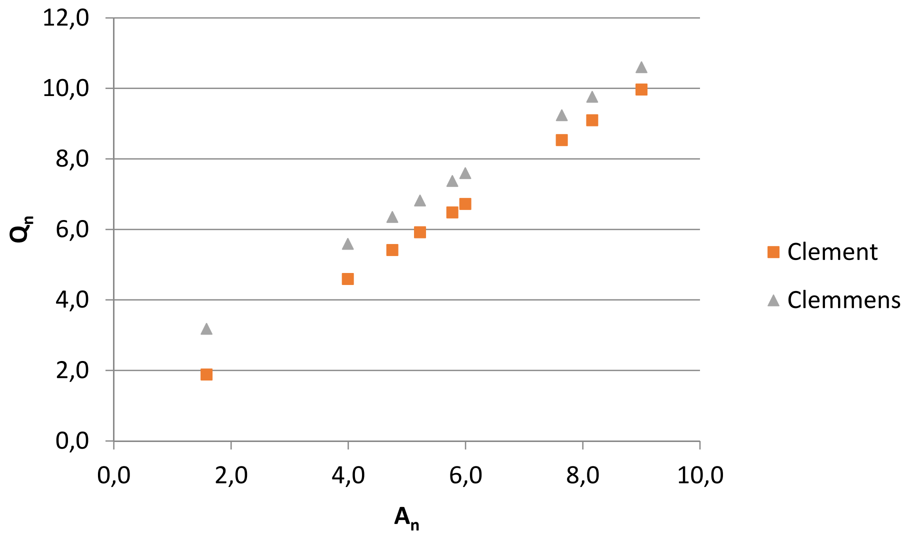

Figure 9. Secondary canal dimensionless discharge (Q_n) related to dimensionless service area (A_n) for Clément's formula and Clemmens' arrange formula for a 95 % service level.

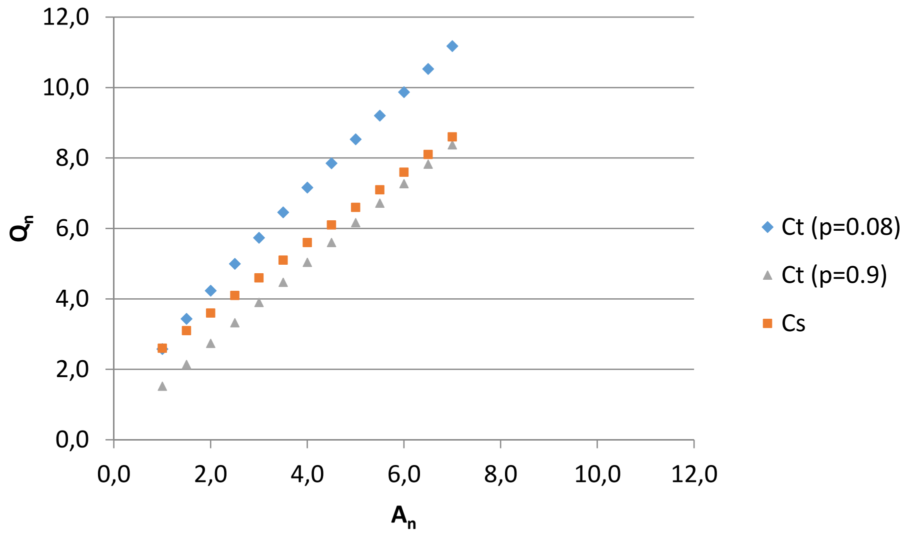

Figure 10. Tertiary canal dimensionless discharge depending on service area, discharge equation (Ct : Clement's formula, Cs : Clemmens' formula) and opening probability (*p*)

As explained, the equation for a secondary canal (Eq. 8) is different than that for a tertiary canal (Eq. 5). Nevertheless, when we adjusted Eq. 5 to our data, an average probability of 0.967 was obtained, which implies that we are in a case of high opening probabilities. Then the abnormal results obtained in Fig. 9 can be explained as in Figure 10.
Another remarkable fact from Figure 10 is that for a service area, tertiary canal capacity is higher when the opening probability is lower. Eq. 10 shows that low probability is related to high plot discharge (*d*) when all the other variables remain constant (q_s, A_f, *r*). Therefore, for a given service area, when the probability is low, the canal's discharge will be higher.

Sensitivity analysis

In this section, the relevant factors that influence canal discharge are analyzed. Assuming that the intake's CV is constant for any q_s and considering Eqs. 8 and 11,

$$Q_c = \sum_{j=1}^{R} q_s\, A_j + U \sqrt{\sum_{j=1}^{R} (CV_j\; q_s\; A_j)^2} \;= q_s \left(A_{Tc} + U \sqrt{\sum_{j=1}^{R} (\,CV_j A_j)^2} \right) \quad (13)$$

Where *R* is the number of canal intakes.
Equation 13 shows that the most relevant factor affecting Q_c was q_s, followed by A_{Tc}. The *U* variable had a lower impact.

Conclusions

The studied secondary canal has an arranged schedule with no discharge restrictions and, therefore, can be considered a quasi-demand schedule.
Clément's first formula fits well with the observed cumulative probability curve of the canal discharge at different sections. Canal's intakes should be characterized by its average discharge and standard deviation and not as turnout with an opening probability. Thus, the

form of Clément’s first formula for secondary canals is different than the one for tertiary canals.
The average intake’s discharge can be calculated by multiplying the specific continuous discharge by the service area. The coefficient of variation of an intakes’ discharge was similar between intakes, except for smaller intakes, which had higher values.
Clemmens’ arranged delivery schedule equation does not apply to this case because the canal delivers pressurized irrigation and high turnout opening probabilities occur.
The sensitivity analysis indicated that the specific unit discharge was the most relevant variable in determining canal discharge, while service area and service level had less impact.

Data Availability

All data used in the present study were provided by a third party. Direct requests for these materials may be made to the provider indicated in the Acknowledgments.

Acknowledgments

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